# Physics 606 — Spring 2015

## Homework 1

*Instructor: Rainer J. Fries* *Turn in your work by February 9*

[1] **$\delta$-function Fourier Integral** (15 points)

Prove that the inverse Fourier transformation of $e^{-iky}/\sqrt{2\pi}$ in the variable $k$ yields the Dirac $\delta$-function $\delta(x-y)$. In other words, show that

$$\frac{1}{2\pi}\int e^{ik(x-y)}dk \tag{1}$$

as a function of $x$ has the same properties as the function $\delta(x-y)$.

[2] **Bohr-Sommerfeld Quantization** (25 points)

Using the quantization criterion given by Bohr and Sommerfeld calculate the possible energies allowed for a particle of mass $m$

(a) in an harmonic oscillator potential $U(x) = \frac{1}{2}kx^2$.

(b) in an infinitely deep 1-D potential well of width $L$, i.e. $U(x) = 0$ for $-L/2 < x < L/2$ and $U(x) \to \infty$ everywhere else.

[3] **Gaussian Wave Packets – Part I** (35 points)

Consider the real-valued 1-D Gauss function

$$f(x) = Ce^{-\frac{(x-x_0)^2}{4\sigma^2}}\,. \tag{2}$$

with real parameters $x_0$ and $\sigma$.

*Note: All integrals in this problem can be done with basic real and complex calculus. Attempt to solve them yourself to receive full credit.*

(a) How does one have to choose the normalization factor $C$ such that the Gaussian has $L^2$-norm 1, i.e.

$$\int_{\mathbb{R}} |f(x)|^2 dx = 1\,? \tag{3}$$

(b) Calculate the expectation value of position and the variance around it, i.e. evaluate

$$\langle x\rangle = \int_{\mathbb{R}} x|f(x)|^2 dx\,, \tag{4}$$

$$(\Delta x)^2 = \langle (x-x_0)^2\rangle = \int_{\mathbb{R}} (x-x_0)^2\,|f(x)|^2 dx\,. \tag{5}$$

What are thus the interpretations of $x_0$ and $\sigma$ in $f$?

(c) Calculate the Fourier transform $\hat{f}(k)$ of the Gauss function. What are the average value and variance? Show that Gaussian wave packets exhaust the inequality of the uncertainty relation, i.e. they have a minimal uncertainty

$$\Delta x \Delta k = \frac{1}{2} \tag{6}$$

[4] **Refresher: A Simple Hamilton-Jacobi Problem** (25 points)

Consider a particle of mass $m$ moving in one dimension with potential energy $U(x) = -bx$. Write down the Hamilton-Jabobi equation, solve for the action $S$ and derive the motion $x(t)$ of the system with initial conditions $x(0) = x_0$, $\dot{x}(0) = v_0$.