Physics 606 — Spring 2014

Homework 3

Instructor: Rainer J. Fries *Turn in your work by February 11*

[1] **Schrödinger Equation in Momentum Space** (25 points)

Starting from the Schrödinger equation in coordinate space for a particle of mass m and potential energy $V(\vec{r})$ explicitly show that

$$i\hbar\frac{\partial}{\partial t}\phi(\vec{p},t) = \frac{p^2}{2m}\phi(\vec{p},t) + V(-i\hbar\nabla_p)\phi(\vec{p},t) \tag{1}$$

where $\phi(\vec{p},t)$ is the Fourier transformation of a coordinate space solution $\psi(\vec{r},t)$.

[2] **Operator Algebra** (25 points)

(a) Let F and G be two operators on a vector space of functions who both commute with their commutator $[F,G]$. Show that for any $n \in \mathbb{N}$

$$[F,G^n] = nG^{n-1}[F,G] \tag{2}$$

$$[F^n,G] = nF^{n-1}[F,G] \tag{3}$$

(b) Prove that the Jacobi identity

$$[F,[G,H]] + [H,[F,G]] + [G,[H,F]] \tag{4}$$

holds for arbitrary operators F, G, H.

[3] **Properties of the Angular Momentum Operator** (25 points)

(a) Calculate the commutator $[\vec{L},T]$ of the angular momentum operator $\vec{L} = \vec{r}\times\vec{p}$ with the kinetic energy operator $T = p^2/2m$ for a Schrödinger field of mass m both in coordinate and momentum space representation.

(b) Show that the equation of motion for the expectation value of the the angular momentum of a Schrödinger field is

$$\frac{d}{dt}\langle L\rangle = \left\langle \vec{r}\times\vec{F}\right\rangle \tag{5}$$

for a system with potential energy $V(\vec{r})$ where $\vec{F} = -\nabla V(\vec{r})$.

[4] **Baker-Campbell-Hausdorff Relations** (25 points)

Let F, G be two operators on a vector space of functions.

(a) Prove that

$$e^F G e^{-F} = \sum_{k=0}^{\infty} \frac{1}{k!} \big[F, \big[F, \ldots, \big[F, G\big]\big] \ldots\big] \tag{6}$$

where the ... indicate k applications of the commutator with G.

(b) Prove the following simplified Baker-Campbell-Hausdorff formula if F and G both commute with their commutator $[F, G]$:

$$e^F e^G = e^{F+G+[F,G]/2} \,. \tag{7}$$