Spin-Isospin Excitations and Muon Capture in Nuclei

V. A. Kuz'min[1] T. V. Tetereva[2]

[1]Joint Institute for Nuclear Research, Dubna, Russia

[2]Skobeltsyn Institute of Nuclear Physics,
Lomonosov Moscow State University, Moscow, Russia

CSSP 2007

Introduction

Why nuclear muon capture is useful for astrophysics problems?

1. Muon capture and weak nuclear current.

$$\frac{1}{ft} \sim |g_V|^2 B(\mathrm{F}) + |g_A|^2 B(\mathrm{GT})$$

   Axial and pseudoscalar couplings.
2. Muon capture and spin-isospin excitations.
   Strength function of Gamow-Teller transitions and nuclear residual interactions.

Ordinary Muon Capture

Nuclear muon capture

$$\mu^- + A(Z,N) \rightarrow B(Z-1,N+1) + \nu_\mu$$

Weak currents. The nucleon current in relativistic form

$$\begin{aligned}\langle N_n|\mathcal{J}^\alpha|N_p\rangle = \quad & i\bar{u}_n \left[g_V(q^2)\gamma^\alpha + \frac{g_M(q^2)}{2M_p}\sigma^{\alpha\beta}q^\beta \right. \\ & \left. - \left(g_A(q^2)\gamma^\alpha + i\frac{g_P(q^2)}{m_\mu}q^\alpha \right)\gamma_5 \right] t^+ u_p \\ & \times \exp\left(i\vec{q}\cdot\vec{r}\right) \exp\left(i(E_n - E_p)t\right), \quad q^\alpha = (p-n)^\alpha,\end{aligned}$$

and as the operators acting on the nuclear wave functions

$$J_k = t^+ \begin{cases} g_V + [g_P - g_A - g_M]\dfrac{E_\nu}{2M_N}(\vec{\sigma},\hat{\vec{\nu}}) + \dfrac{g_A}{M_N}\left(\vec{p},\vec{\sigma}\right), & \text{“time” part} \\ \left(g_A - [g_V + g_M]\dfrac{E_\nu}{2M_N}\right)\vec{\sigma} + \dfrac{g_V}{M_N}\vec{p}, & \text{“space” part} \end{cases}$$

The lepton current components are

$$\left.\begin{array}{ll} \chi_\nu^\dagger(r)\left(1-(\vec{\sigma},\hat{\vec{\nu}})\right)\chi_\mu(r), & \text{“time” part} \\ \chi_\nu^\dagger(r)\left(1-(\vec{\sigma},\hat{\vec{\nu}})\right)\vec{\sigma}\chi_\mu(r), & \text{“space” part} \end{array}\right\} \sim \exp\left[i\left(\vec{\nu},\vec{r}\right)\right],$$

respectively.

Weak nucleon couplings and total rates of OMC

Advantages over beta-decay.

- No energy limitations (the *window of beta-decay* absent).
- OMC rate $\Lambda_k \sim E_\nu^2$, which is

$$E_\nu \approx m_\mu \left\{ 1 - \frac{M_f + E_k + \epsilon_{1S} - M_i}{m_\mu} \right\},$$

therefore

$$\frac{\Delta\Lambda_k}{\Lambda_k} \sim \frac{\Delta E_k}{E_\nu} \ll \frac{\Delta E_k}{E_k}$$

as an result — relaxed requirements to the precision of nuclear structure calculations. *Random Phase Approximation* will be enough.

Nuclear model Hamiltonian.

$$H_{\mathrm{M}} = \sum_{\tau=\mathrm{n,p}} H_0(\tau) + H_{\mathrm{res}},$$

$$H_0(\tau) = \sum_{j_\tau, m_\tau} E_{j_\tau} a^\dagger_{j_\tau, m_\tau} a_{j_\tau, m_\tau}$$

$$- \frac{G_\tau}{4} \sum_{j'_\tau, m'_\tau, j_\tau m_\tau} (-)^{j'_\tau - m'_\tau + j_\tau - m_\tau} a^\dagger_{j'_\tau, m'_\tau} a^\dagger_{j'_\tau, -m'_\tau} a_{j_\tau, -m_\tau} a_{j_\tau, m_\tau},$$

$$H_{\mathrm{res}} = -\frac{1}{2} \sum_{J,M} (\kappa_0^J + \kappa_1^J (\vec{\tau}_1 \cdot \vec{\tau}_2)) Q^\dagger_{JM}(1) Q_{JM}(2)$$

$$- \frac{1}{2} \sum_{L,J,M} (\kappa_0^{LJ} + \kappa_1^{LJ} (\vec{\tau}_1 \cdot \vec{\tau}_2)) Q^\dagger_{LJM}(1) Q_{LJM}(2)$$

$$Q_{JM} = \sum_{jmt_3, j'm't'_3} \left\langle j'm't'_3 | i^J f_J Y_{JM} \tau^k | jmt_3 \right\rangle a^\dagger_{j'm't'_3} a_{jmt_3}$$

$$Q_{LJM} = \sum_{jmt_3, j'm\prime t'_3} \left\langle j'm't'_3 | i^L f_{LJ} [Y_L, \sigma]_{JM} \tau^k | jmt_3 \right\rangle a^\dagger_{j'm't'_3} a_{jmt_3}$$

Total rates of OMC in Zr and Mo.
The $\Lambda_{\mathrm{tot}}^{\mathrm{OMC}}\left({}^{90}\mathrm{Zr}\right)$ (in $10^5\,\mathrm{s}^{-1}$) calculated with different sets of the constants of effective nuclear residual interactions.

$\frac{g_P}{g_A}$	$\kappa_1^{LJ} =$ 0.0 / $\kappa_1^{J} =$ 0.0	-0.23 / -0.42	-0.33 / -0.42	-0.43 / -0.42	-0.33 / -0.21	-0.33 / -0.63
2.0	137.9	95.99	90.37	86.89	91.31	90.33
4.0	132.8	92.42	87.06	83.73	88.00	87.02
6.0	128.3	89.29	84.15	80.95	85.08	84.11
8.0	124.5	86.59	81.63	78.54	82.57	81.59
10.0	121.3	84.33	79.51	76.50	80.45	79.47
12.0	118.7	82.50	77.78	74.84	78.72	77.75
14.0	116.7	81.10	76.46	73.54	77.39	76.42

Total rates of ordinary muon capture (in $10^5\,\mathrm{s}^{-1}$) for different values of g_P/g_A and **free** value of $g_A = -1.26 g_V$.

g_P/g_A	${}^{\mathrm{nat}}\mathrm{Zr}$	${}^{\mathrm{nat}}\mathrm{Mo}$
2.0	86.9	102.9
4.0	83.7	99.2
6.0	80.9	96.0
8.0	78.5	93.2
10.0	76.5	91.0
12.0	74.8	89.2
14.0	73.6	87.8
Experiment	86.6 ± 0.8	96.1 ± 1.5

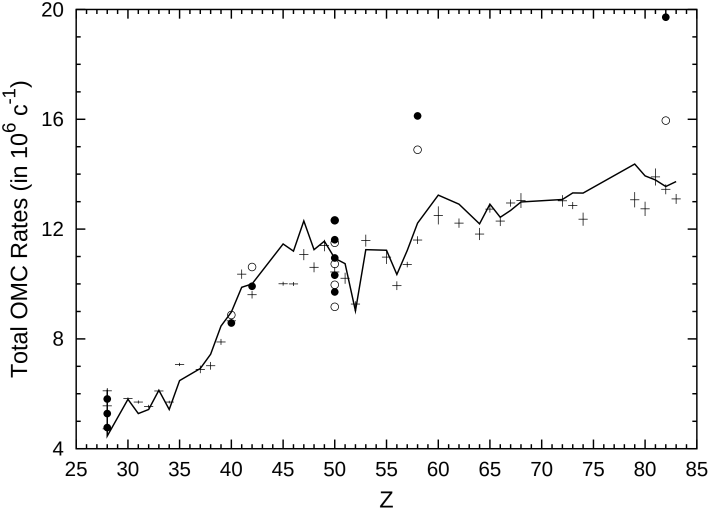
Experiment: T. Suzuki, D. F. Measday, and J. P. Roalsvig, Phys. Rev. C **35,** 2212 (1987).

The γ-yield in Radiative Muon Capture in Nuclei

$$\mu^- + A(Z,N) \rightarrow B(Z-1, N+1) + \gamma + \nu_\mu, \qquad (E_\gamma \geq 57\,\mathrm{MeV})$$

Target	g_P/g_A	Γ^{RMC} (in s^{-1})		$\frac{\Gamma^{\mathrm{RMC}}}{\Lambda^{\mathrm{OMC}}_{\mathrm{tot}}}$ (in 10^{-5})		
		IA	MIA	IA	MIA	Experiment
$^{\mathrm{nat}}\mathrm{Zr}$	2.0	192.4	122.3	2.21	1.41	1.31 ± 0.15
	6.0	258.6	129.5	3.20	1.60	
	10.0	363.3	147.1	4.75	1.92	
	14.0	506.5	175.1	6.88	2.38	
$^{\mathrm{nat}}\mathrm{Mo}$	2.0	234.1	149.4	2.28	1.45	1.11 ± 0.11
	6.0	317.2	154.8	3.31	1.61	
	10.0	448.7	173.2	4.93	1.90	
	14.0	628.6	204.4	7.16	2.33	

General picture: Comparison between experimental and calculated total capture rates



Short conclusions.

- For medium size nuclei the calculations with **free** values of weak axial coupling describe well the total rates of OMC. There is no clear dependency on the residual nuclear interaction.
- For heavy nuclei the theoretical total OMC rates exceed the experimental ones. Theoretical rates are sensitive to the nuclear residual interaction.
- The rates of RMC are sensitive to the induced pseudoscalar coupling, g_P, but theory and experiment differ too high. Future investigations are necessary.

Gamow-Teller Transitions

- Charge exchange-nuclear reaction at intermediate energies. For (p, n) reaction

$$\frac{d\sigma}{d\omega}(0^\circ) = \frac{\mu}{\pi\hbar^2}\frac{k_f}{k_i}\left[V_\tau J_\tau^2 B^-(\mathrm{F}) + V_{\sigma\tau} J_{\sigma\tau}^2 B^-(\mathrm{GT})\right].$$

- Reduced probabilities of Gamow-Teller transitions

$$B^\pm(\mathrm{GT}) \equiv \frac{1}{2J_i+1}\sum_{M_i,M_f}\sum_{m=-1}^{1}\left|\langle J_f M_f|\sum_{q=1}^{A}\sigma_q^m\, t_q^\pm|J_i M_i\rangle\right|^2,$$

$$|n\rangle = t^+|p\rangle$$

- Ikeda's sum rule and problem of *missed* GT strength

$$\sum_k B_k^-(\mathrm{GT}) - \sum_\ell B_\ell^+(\mathrm{GT}) = 3(N-Z)$$

Only 60% of theoretical $\sum_k B_k^-(\mathrm{GT})$ were experimentally found in the region of GT resonance and below it.

Missed strength and residual interactions

The main properties of Random Phase Approximation. (RPA).
Phonon operators

$$\Omega_\rho = \sum_{p,h} \left(\psi^\rho_{ph} a^\dagger_h a_p - \phi^\rho_{ph} a^\dagger_p a_h \right)$$

Phonon amplitudes are determined by

$$\begin{aligned} \langle \mathrm{HF} | \left[[\Omega_\rho, H]_- , a^\dagger_p a_h \right]_- | \mathrm{HF} \rangle &= E_\rho \langle \mathrm{HF} | \left[\Omega_\rho, a^\dagger_p a_h \right]_- | \mathrm{HF} \rangle, \\ \langle \mathrm{HF} | \left[[\Omega_\rho, H]_- , a^\dagger_h a_p \right]_- | \mathrm{HF} \rangle &= E_\rho \langle \mathrm{HF} | \left[\Omega_\rho, a^\dagger_h a_p \right]_- | \mathrm{HF} \rangle. \end{aligned}$$

For any one-body operator R the amplitude of transition from the ground state is

$$R_\rho \equiv \langle \rho | R | 0 \rangle_{\mathrm{RPA}} \equiv \langle \mathrm{HF} | [\Omega_\rho, R]_- | \mathrm{HF} \rangle$$

Mathematical properties

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} \psi^\rho \\ \phi^\rho \end{pmatrix} = E_\rho \begin{pmatrix} \psi^\rho \\ -\phi^\rho \end{pmatrix}$$

with

$$A_{ph,p'h'} = \langle \mathrm{HF}|a_h^\dagger a_p H a_{p'}^\dagger a_{h'}|\mathrm{HF}\rangle - \langle \mathrm{HF}|H|\mathrm{HF}\rangle$$
$$B_{ph,p'h'} = \langle \mathrm{HF}|a_h^\dagger a_p a_{h'}^\dagger a_{p'} H|\mathrm{HF}\rangle$$

If A is positive-definite

$$\langle \mathrm{HF}| \left[\Omega_\alpha, \Omega_\beta^\dagger\right]_- |\mathrm{HF}\rangle = \mathrm{sign}(E_\alpha)\delta_{\alpha,\beta}.$$

The positive E_ρ's are the excitation energy of the system. As a phonon operator is one-body operator for positive E_β

$$\langle \alpha|\Omega_\beta^\dagger|0\rangle_{\mathrm{RPA}} \equiv \langle \mathrm{HF}| \left[\Omega_\alpha, \Omega_\beta^\dagger\right]_- |\mathrm{HF}\rangle = \delta_{\alpha,\beta},$$

which is the orthonormality condition for one-phonon states.

Sum rules.
For any positive integer k and any one-body operator R

$$\sum_{\rho:E_\rho>0} E_\rho^k \left(|R_\rho|^2 - (-1)^k |R_\rho^\dagger|^2\right) =$$

$$= \begin{pmatrix} R^\dagger & \widehat{R^+} \end{pmatrix} \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}^k \begin{pmatrix} R \\ -R^{+*} \end{pmatrix},$$

here (R) and R^+ are column vectors consisted of matrix elements $\langle \mathrm{HF}|a_h^\dagger a_p R|\mathrm{HF}\rangle$ and $\langle \mathrm{HF}|a_h^\dagger a_p R^\dagger|\mathrm{HF}\rangle$ respectively.

Interaction between $1p{-}1h$ and $2p{-}2h$ configurations.

- Second RPA

$$\mathcal{O}^\dagger_\rho \sim \ldots + \sum_{p<p',h<h'} \left(\psi^\rho_{pp',hh'} a^\dagger_p a^\dagger_{p'} a_{h'} a_h - \phi^\rho_{pp',hh'} a^\dagger_h a^\dagger_{h'} a_{p'} a_p \right)$$

- Fragmentation problem

$$|JM\rho\rangle = \left(R_{\rho_1\rho} \Omega^\dagger_{JM\rho_1} + P_{\rho_1\rho_2\rho} \left[Q^\dagger_{J_1\rho_1} \otimes \Omega^\dagger_{J_2\rho_2} \right]_{JM} \right) |\rangle$$

- Strength function

$$b^{\pm}(E) = \sum_k \delta(E - E_k) B_k^{\pm}(\mathrm{GT})$$

  and its energy-weighted moments

$$S_m^{\pm} = \int_0^{\infty} b^{\pm}(E) E^m dE .$$

- Exact results

  Zero- and first order sum rules for any one-body transitional operator calculated within both RPA and SRPA are equal each other.

Second RPA

$$S_0^- - S_0^+ \Big|_{\mathrm{SRPA}} = S_0^- - S_0^+ \Big|_{\mathrm{RPA}}$$

$$S_1^- + S_1^+ \Big|_{\mathrm{SRPA}} = S_1^- + S_1^+ \Big|_{\mathrm{RPA}}$$

Fragmentation problem

$$S_0^- \Big|_{\mathrm{fragm.}} = S_0^- \Big|_{\mathrm{RPA}}$$

$$S_1^- \Big|_{\mathrm{fragm.}} = S_1^- \Big|_{\mathrm{RPA}}$$

Due to neutron excess $S_0^+ \ll S_0^-$ in heavy nuclei.

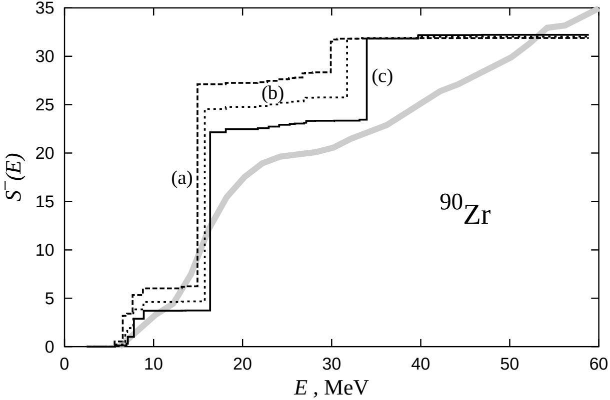
Finally:
$$S_0^-\big|_{2p-2h} \approx S_0^-\big|_{1p-1h}$$
$$S_1^-\big|_{2p-2h} \approx S_1^-\big|_{1p-1h}$$

Average excitation energies which is S_1^-/S_0^- calculated within $1p{-}1h$ space, only, and taking into account the interaction of $1p{-}1h$ configurations with $2p{-}2h$ and more complicated ones should be approximately equal (or equal as in fragmentation problem) to each other: $\langle E \rangle|_{2p-2h} \approx \langle E \rangle|_{1p-1h}$

Conclusion: It is necessary for describing the effect of missed GT strength that the residual interaction between $1p{-}1h$ configurations should shift a large part of total transition strength to the higher excitation energies.

A nuclear residual interaction should have a specific feature: It must mix $\Delta N = 0$ and $\Delta N \geq 2$ particle-hole configurations.

Example. GT transitions in $^{90}\mathrm{Zr}$.



Distribution of the strength of σt^{-} operator calculated for several values of effective constant of spin-isospin residual interaction. The thick gray curve is the distribution extracted from the cross-sections of $^{90}\mathrm{Zr}(p,n)^{90}\mathrm{Nb}$ reaction (T. Wakasa, *et al.,* Phys. Rev. C **55,** 2909 (1997)).

$^{208}\mathrm{Pb}$

The variants of residual interactions.

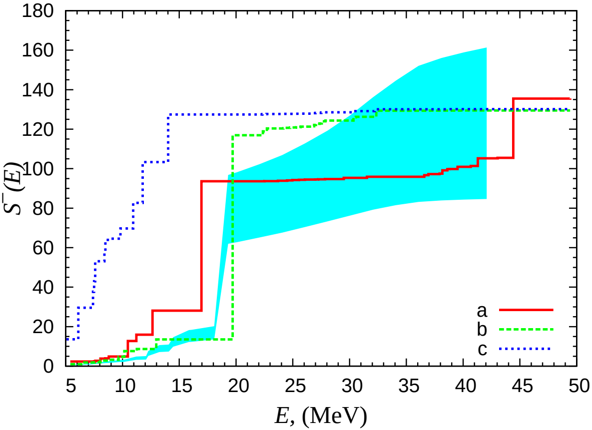
We consider the results of calculations with two different residual interaction ("a" and "b" in the table) and without residual interaction ("c").

	The residual interaction		Without residual interaction c
	Mixes $\Delta N = 0$ and $\Delta N \geq 2$ configurations a	Not mixes $\Delta N = 0$ and $\Delta N \geq 2$ configurations b	
S_0^+	6.55	0.49	1.15
S_0^-	135.55	129.49	130.15
$S_0^- - S_0^+$	129.00	129.00	129.00

σt^- strength, as in ${}^{208}\mathrm{Pb}(p,n){}^{208}\mathrm{Bi}$ reaction.

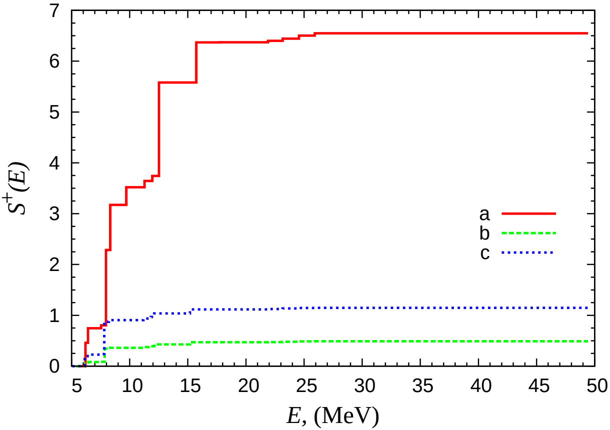
$$S^-(E) = \sum_{k:E_k \le E} B_k^-(\mathrm{GT}) = \int_0^E b^-(t)dt$$

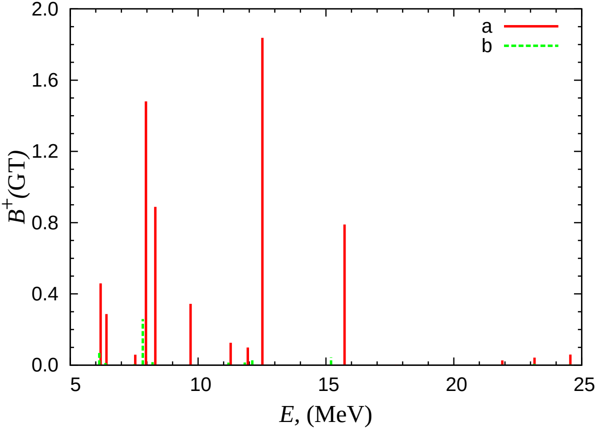
The energies are measured from the ground state of target nucleus.



Experiment: B. S. Flanders *et al.*, Phys. Rev. C **40,** 1985 (1989).

σt^+ strength, as in $^{208}\mathrm{Pb}(n,p)^{208}\mathrm{Tl}$ reaction.

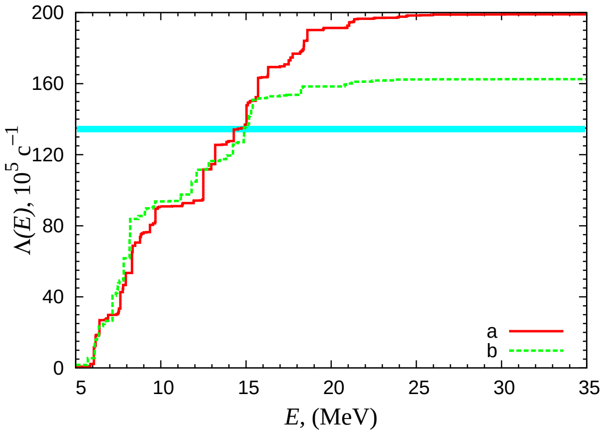




a
b
2.0
1.6
1.2
0.8
0.4
0.0
5
10
15
20
25
B^+(GT)
E, (MeV)

Total rates of muon capture $^{208}\mathrm{Pb}(\mu,\nu)^{208}\mathrm{Tl}$

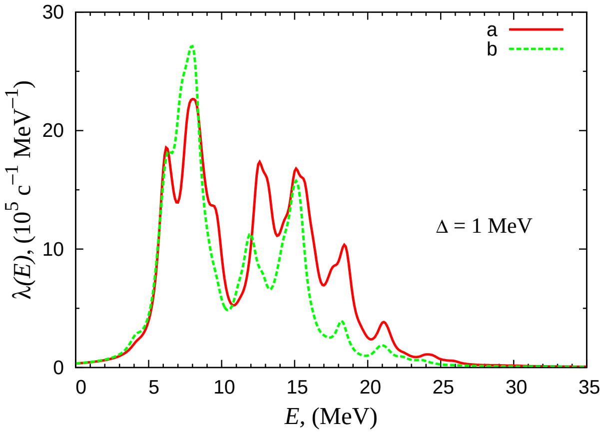
$$\Lambda(E) = \sum_{k:E_k \le E} \Lambda_k$$



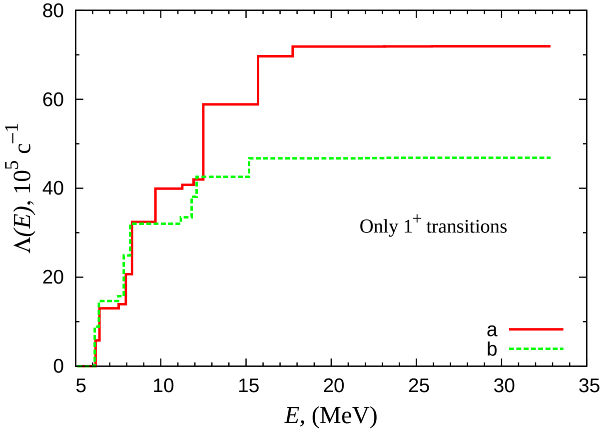
Experiment: T. Suzuki, D. F. Measday, and J. P. Roalsvig, Phys. Rev. C **35,** 2212 (1987)

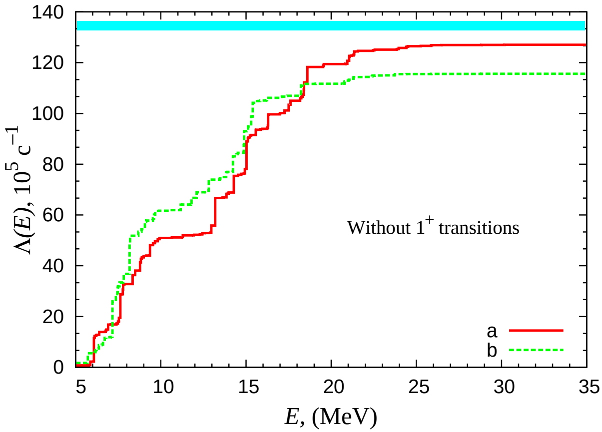
Rates of muon capture $^{208}\mathrm{Pb}(\mu,\nu)^{208}\mathrm{Tl}$

$$\lambda(E) = \sum_k \rho(E - E_k)\Lambda_k, \quad \rho(E - E_k) = \frac{\Delta}{2\pi}\frac{1}{(E - E_k)^2 + \Delta^2/4}.$$



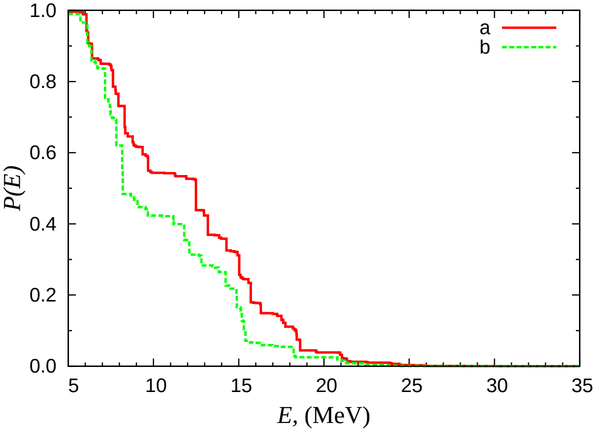
Rates of muon capture $^{208}\mathrm{Pb}(\mu,\nu)^{208}\mathrm{Tl}$





140
120
100
80
60
40
20
0
5
10
15
20
25
30
35
$\Lambda(E)$, 10^5 c^{-1}
E, (MeV)
Without 1^+ transitions
a
b

$$P(E) = 1 - \sum_{k:E_k \leq E} \Lambda_k / \Lambda_{\text{tot}}$$



The question is: How can one experimentally feel the difference between the curves in the energy range $15\,(\mathrm{MeV}) \leq E \leq 20\,(\mathrm{MeV})$?

Estimations for the probability of neutron emission in $^{208}\mathrm{Pb}(\mu, \nu\, kn)^{208-k}\mathrm{Tl}$.

Under assumption that neutron will leave the nucleus each time when it has enough energy (all decay channels except the neutron one are neglected), we can estimate the probability distribution of multiplicity of neutrons released after muon capture.

Interaction or $S_{k\,n}$	Probability of emission k neutrons			
	$k = 0$	$k = 1$	$k = 2$	$k \geq 3$
a	38	44	17	1
b	55	38	6	1
$S_{k\,n}$, (MeV)		9.305	16.147	22.650
Experiment on $^{209}\mathrm{Bi}$	5	47	29	19
$S_{k\,n}$, (MeV)		5.092	12.459	19.197

Experiment: D. F. Measday, T. J. Stocki, and H. Tan, Phys. Rev. C **75**, 045501 (2007).

Conclusions

1. The calculated pattern of neutron multiplicity in reaction $^{208}\mathrm{Pb}(\mu, \nu\, kn)^{208-k}\mathrm{Tl}$ depends on the nuclear spin-isospin residual interaction in particle-hole channel.
2. The experimental pattern will be a strong test for nuclear spin-isospin residual interaction, and therefore for the effect of missed Gamow-Teller strength.