Superallowed Fermi beta decay: the radiative correction

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Precision studies of superallowed beta decay are used to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ud} and to study the matrix's unitarity. To reach this goal, however, the experimental β-decay rates need to be corrected for radiative and isospin-symmetry-breaking effects. The radiative correction, which in total is around 4%, is conveniently divided into terms that are nucleus dependent, the 'outer' radiative correction δ_R, and terms that are nucleus independent, the 'inner' radiative correction Δ_R. The outer correction is fairly unambiguous and is given by a QED calculation. The inner correction is more troublesome. Since nuclear β decay involves hadronic states, uncertainties arise from the presence of strong interactions. To minimize these effects, attention is focused on superallowed transitions between 0^+ states where the conserved vector current (CVC) hypothesis protects the decay amplitudes at the tree level from strong-interaction corrections. However, radiative corrections go beyond the tree level and introduce loop corrections that can involve the weak axial-vector interaction that is not protected by CVC. These effects require a model-dependent calculation.

At the time of our 2005 survey [1], the inner radiative correction was taken from [2] to be

$$\Delta_R = (2.40 \pm 0.08)\%, \tag{1}$$

where the error was an estimate of the uncertainty of the axial-vector contribution to the loop diagram. This uncertainty in fact dominated the accuracy with which V_{ud} could be determined from nuclear β decays. In our survey we quoted:

$$|V_{ud}|^2 = 0.9482 \pm 0.0008. \tag{2}$$

The total quoted uncertainty, which is 0.00083 if the next significant figure is included, is dominated by the uncertainty arising from Δ_R, *viz.* 0.00074.

In 2006, Marciano and Sirlin [3] reevaluated the loop correction paying particular attention to the causes of the uncertainty. They divided the loop integration into three energy regimes. At low energy the interactions are with structureless nucleons (but with nucleus-modified coupling constants) and the loop diagram reduces to the easily evaluated Born graphs. At the other extreme of high energies the interactions are with quarks yielding a leading-log term and QCD corrections, which are related to a Bjorken sum rule and hence are under control. At intermediate energies Marciano and Sirlin introduced an interpolation function inspired by vector meson dominance models and matched it to the low-energy and high-energy loop contributions. By assuming a 100% error in the interpolator and a 10% error in the low-energy contribution, Marciano and Sirlin [3] reduced the uncertainty on Δ_R by a factor of two, from 0.08% (see Eqn. 1) to 0.038%.

We have now adopted these new calculations into our analysis of the superallowed β transitions but in doing so we have moved one small term from the inner to the outer correction. This reorganization is prompted by the estimate of Czarnecki, Marciano and Sirlin [4] of the order α^2 correction. Their estimate is based on a renormalization-group analysis of the leading log extrapolation: in the high-energy regime, this extrapolation remains an inner correction; but at low energies, the leading log depends on the maximum electron energy and is thus nucleus dependent. Consequently we place its extrapolation in the outer correction. This leads to the result we now use for the inner radiative correction:

$$\Delta_R = (2.361 \pm 0.038)\%. \tag{3}$$

At the same time, we must of course update our outer radiative corrections to incorporate the transferred contributions. Our division of the total radiative correction, *RC*, now is

$$\mathrm{RC} = \delta_R + \Delta_R$$
$$\text{where } \delta_R = \delta_R' + \delta_{\alpha^2} + \delta_{NS}. \tag{4}$$

Here δ_R' has our usual definition [5] and includes the corrections to order α, $Z\alpha^2$ and $Z^2\alpha^3$. The second component of δ_R incorporates the new order-α^2 corrections, and the third, δ_{NS}, is the nuclear-structure dependent term, which comes from the low-energy Born graphs and is slightly changed by the new reorganized arrangement.

In Table I we give some sample results, comparing 2005 values for the radiative corrections with today's values. The new values for the total radiative corrections, *RC*, differ from the 2005 values by 0.01 to 0.02%. This shift is very small and has no impact on the central value of V_{ud}. The reduction in the error, on the other hand, gives a significant improvement in the accuracy of V_{ud} and in the sharpness of the CKM unitarity test.

Table I. Contributions to the radiative correction in percent units.

Parent	2005					2006				
	δ_R'	δ_{α^2}	δ_{NS}	Δ_R	RC	δ_R'	δ_{α^2}	δ_{NS}	Δ_R	RC
^{10}C	1.652	0.000	-0.360	2.400	3.692	1.652	0.027	-0.357	2.361	3.683
$^{26}Al^m$	1.458	0.000	0.009	2.400	3.867	1.458	0.020	0.012	2.361	3.851
^{54}Co	1.428	0.000	-0.029	2.400	3.799	1.428	0.015	-0.025	2.361	3.779
^{74}Rb	1.485	0.000	-0.065	2.400	3.820	1.485	0.013	-0.061	2.361	3.798

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