Symmetry energy, temperature, density and isoscaling parameter as a function of excitation energy in A ~ 100 mass region

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Continuing our investigation on the density dependence of the symmetry energy from previous years, we have now investigated the same from an alternate point of view – the statistical model approach. Previously, we had studied the density dependence of the symmetry energy by comparison to the dynamical model Antisymmetrized Molecular Dynamic (AMD). The present statistical model approach combines the features of the Statistical Multifragmentation Model (SMM) and the expanding Fermi gas model to obtain a systematic correlation between the temperature, density and symmetry energy of a multifragmenting system as it evolves with the excitation energy. The advantage of this approach is that unlike the dynamical model approaches, where the effect of sequential decay on isoscaling parameter remains controversial, the sequential decay effect in statistical models are well established and known to have negligible effect.

To build the correlation between the temperature, density, and symmetry energy of a multifragmenting system as it evolves with the excitation energy, we have made use of the fragment yield distributions measured in ^{58}Ni, ^{58}Fe + ^{58}Ni, ^{58}Fe reactions at 30, 40 and 47 MeV/nucleon. The yield distributions were used to obtain the isoscaling parameter α, as a function of the excitation energy of the fragmenting source. The parameter α was obtained from the ratios of the isotopic yields for two different pairs of reactions, ^{58}Fe + ^{58}Ni and ^{58}Ni + ^{58}Ni, and ^{58}Fe + ^{58}Fe and ^{58}Ni + ^{58}Ni. Fig. 1(a) shows the experimental isoscaling parameter α as a function of the excitation energy for Fe + Fe and Ni + Ni, and Fe + Ni and Ni + Ni pairs of reactions. A systematic decrease in the absolute values of the isoscaling parameter with increasing excitation energy is observed for both pairs. The α parameters for the ^{58}Fe + ^{58}Fe and ^{58}Ni + ^{58}Ni are about twice as large compared to those for the ^{58}Fe + ^{58}Ni and ^{58}Ni + ^{58}Ni pair of reactions. The experimental isoscaling parameters were compared with the predictions of the Statistical Multifragmentation Model (SMM) to study their dependence on the excitation energy and the isospin content. The break-up density in the calculation was taken to be multiplicity-dependent and was varied from approximately 1/2 to 1/3 the saturation density. This was achieved by varying the free volume with the excitation energy. The form of the dependence was adopted from the work of Bondorf *et al.* [1,2], (and shown by the solid curve in Fig. 1(d)). It is known that the multiplicity-dependent break-up density, which corresponds to a fixed inter-fragment spacing and constant pressure at break-up, leads to a pronounced plateau in the caloric curve [1,2]. A constant break-up density would lead to a steeper temperature versus excitation energy dependence. The symmetry energy in the calculation was then varied until a reasonable agreement between the calculated and the measured α was obtained. Fig. 1(a) shows the comparison between the SMM calculated and the measured α for both pairs of systems. The dashed curves correspond to the calculation for the primary fragments and the solid curves to the secondary fragments. The width in the curve is the measure of the uncertainty in the inputs to the SMM calculation.

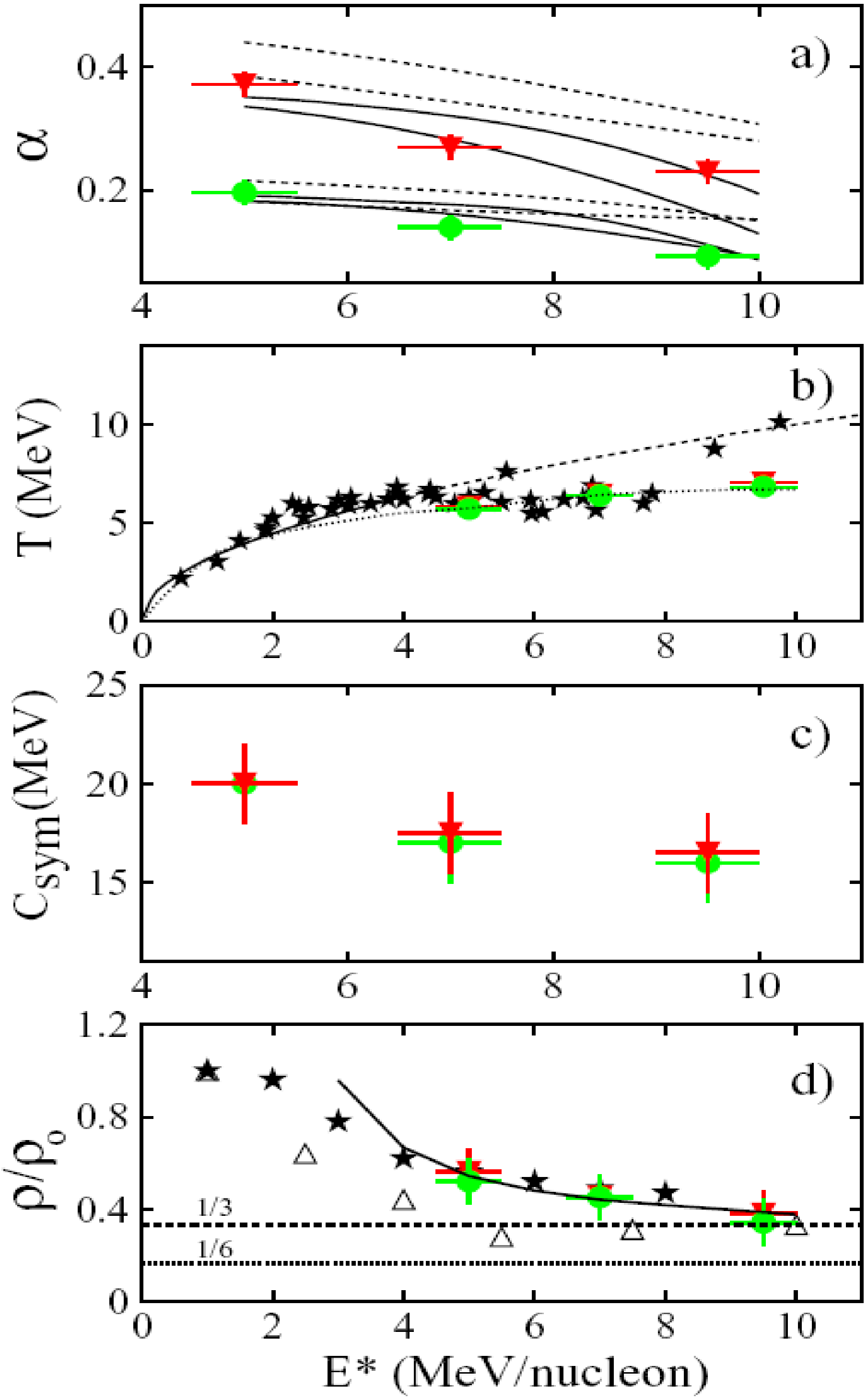

Figure 1. Isoscaling parameter α, temperature, symmetry energy and density as a function of excitation energy for the ^{58}Fe + ^{58}Fe and ^{58}Ni + ^{58}Ni (inverted triangles), and ^{58}Fe + ^{58}Ni and ^{58}Ni + ^{58}Ni (circles) reactions. a) Experimental α as a function of excitation energy. The solid and the dashed curves are the SMM calculations. b) Temperature as a function of excitation energy. The solid stars correspond to the measured values taken from the literature. The solid-and-dashed curve corresponds to the Fermi-gas relation. c) Symmetry energy as a function of excitation energy. d) Density as a function of excitation energy.

Fig. 1(b) shows the temperature as a function of excitation energy (caloric curve) obtained from the above SMM calculation that uses the excitation energy dependence of the break-up density to explain the observed isoscaling parameters. These are shown by the solid and inverted triangle symbols. Also shown in the figure are the experimentally measured caloric curve data compiled by Natowitz *et al.* [3], from various measurements for this mass range. The data from these measurements are shown collectively by solid star symbols and no distinction is made among them. The Fermi-gas model predictions with inverse level density parameter $K_o = 10$ (solid-and-dashed curve) is also shown. It is evident from the figure that the temperatures obtained from the SMM calculations are in good agreement with the overall trend of the caloric curve. The symmetry energies obtained from the statistical model comparison of the experimental isoscaling parameter α, are shown in Fig. 1(c). A steady decrease in the symmetry energy with increasing excitation energy is observed for both pairs of systems.

The phase diagram of the multifragmenting system is two dimensional and hence the excitation energy dependence of the temperature (the caloric curve) must take into account the density dependence too. Often this dependence is neglected while studying the caloric curve. It has been shown by Sobotka *et al.* [4], that the plateau in the caloric curve could be a consequence of the thermal expansion of the system at higher excitation energy and decreasing density. By assuming that the decrease in the breakup density, as taken in the present statistical multifragmentation calculation, can be approximated by the expanding Fermi gas model, the density as a function of excitation energy was extracted using the relation,

$$T^2 = K_o(\rho/\rho_o)^{2/3} E^*$$

In the above expression, the momentum and the frequency dependent factors in the effective mass ratio were taken to be one as is expected at the high excitation energies and low densities studied in this work.

The resulting densities for the two pairs of systems are shown in Fig. 1(d) by the solid circles and inverted triangles. For comparison, the figure also shows the break-up densities obtained from the analysis of the apparent level density parameters required to fit the measured caloric curve by Natowitz *et al.* [3], and those obtained by Viola *et al.* [5], from the Coulomb barrier systematics of the measured intermediate mass fragment kinetic energy spectra. One observes that the present results, obtained by requiring a fit to both the measured isoscaling parameters and the caloric curve, are in good agreement with those obtained by Natowitz *et al*. The figure also shows the fixed freeze-out density of 1/3 (dashed line) and 1/6 (dotted line) of the saturation density assumed in various statistical model comparisons. The caloric curve obtained using the above densities and excitation energies (shown by solid stars, circles and the triangles) with $K_o = 10$ in the above equation, is shown by the dotted curve in Fig. 1(b). The small discrepancy between the dotted curve and the data (solid stars) below 4 MeV/nucleon is due to the approximate nature of the equation being used. It is evident from Figure 1(a), (b), (c) and (d) that the decrease in the experimental isoscaling parameter α, symmetry energy, break-up density, and the flattening of the temperature with increasing excitation energy are all correlated. One can thus conclude that the expansion of the system during the multifragmentation process leads to a decrease in the isoscaling parameter, a decrease in the symmetry energy and density, and the flattening of the caloric curve.

From the above correlation between the symmetry energy as a function of excitation energy and the density as a function of excitation energy, we have obtained the symmetry energy as a function of density. This is shown by the inverted triangles and solid circles in Fig. 2 for the ^{58}Fe + ^{58}Fe ^{58}Ni + ^{58}Ni, and the ^{58}Fe + ^{58}Ni ^{58}Ni + ^{58}Ni pair of reactions. The solid curve in Fig. 2 corresponds to the dependence $C_{sym}(\rho) = 31.6(\rho/\rho_o)^{0.69}$ MeV, obtained from the dynamical Antisymmetrized Molecular Dynamic (AMD) calculation, as discussed in our previous work [6].

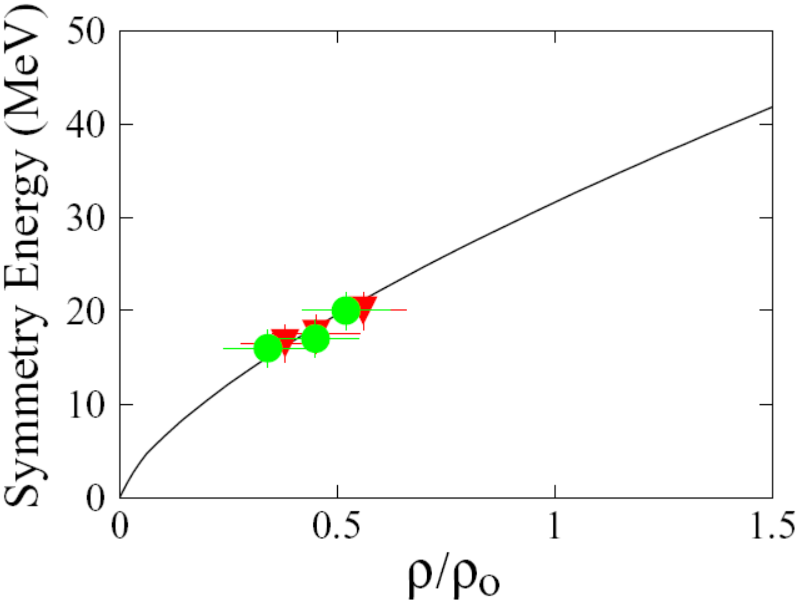

Figure 2. Symmetry energy as a function of density for the ^{58}Fe + ^{58}Fe and ^{58}Ni + ^{58}Ni pair of reaction (inverted triangles), and ^{58}Fe + ^{58}Ni and ^{58}Ni + ^{58}Ni pair of reactions (solid circles) or the 30, 40 and 47 MeV/nucleon. The solid curve is the dependence obtained from the dynamical model analysis.

In the past, attempts have been made to study the density dependence of the symmetry energy by looking at specific observables and comparing them with the predictions of the dynamical models. Such an approach attempts to explain the observable of interest without trying to simultaneously explain other properties, such as, the temperature, density and excitation of the fragmenting system. This has lead to a variety of different dependences without accurate knowledge of what density is being probed. While it might not be straightforward to distinguish different equations-of-state using dynamical models, due to uncertainties in the sequential decay effects, the allure of extracting information on the symmetry energy from the point of view of the basic nucleon-nucleon interaction is very appealing. On the other hand, the determination of the density dependence of the symmetry energy from statistical model analysis by simultaneously explaining the isoscaling parameter, caloric curve and the density as a function of excitation energy is a reverse approach. This approach attempts to explain the experimental observables without any prior knowledge of the governing interaction and arrives at a dependence which can then be compared with those predicted from the basic interactions.

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