Density dependence of the symmetry energy and the nuclear equation of state : A dynamical and statistical model perspective

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The density dependence of the symmetry energy is a key unknown in the equation of state of isospin asymmetric nuclear matter. Due to its direct relevance to the structure and stability of systems as diverse as the neutron stars and neutron-rich nuclei, there is a significant interest in determining this quantity. Theoretical studies based on microscopic "ab-initio" calculations predict a variety of different forms of the density dependence of the symmetry energy. Experimental measurements on the density dependence of the symmetry energy are still scarce and difficult. In the past, we have tried to study the density dependence of the symmetry energy using the fragment isotope yields in multifragmentation reactions compared to the dynamical model Antisymmetrized Molecular Dynamic (AMD). We have now investigated the same using the statistical model approach of multifragmentation reaction. We report here a comparison between the two approaches along with several other independent studies to obtain a constraint on the form of the density dependence of the symmetry energy. We also report several predictions that follow from the constraint.

Fig. 1 shows the density dependence of the symmetry energy obtained from the statistical model approach using the experimentally measured isoscaling parameter (red and green circle symbols) [1]. The green solid curve corresponds to the density dependence of the symmetry energy obtained from the Gogny-AS interaction in our previous analysis using the dynamical AMD approach [2,3], assuming the sequential decay effect to be small. The red dashed curve corresponds to the one obtained from an accurately calibrated relativistic mean field interaction, used for describing the Giant Monopole Resonance (GMR) in ^{90}Zr and ^{208}Pb, and the IVGDR in ^{208}Pb by Piekarewicz *et al.* [4]. The light-blue dashed curve correspond to the one used to explain the isospin diffusion results of NSCL-MSU using the isospin dependent Boltzmann-Uehling-Uhlenbeck (IBUU) model by Tsang *et al.* [5]. The blue dot-dashed curve also corresponds to the one used for explaining the isospin diffusion data of NSCL-MSU by Chen *et al.* [6], but with the momentum dependence of the interaction included in the IBUU calculation. This dependence has been further modified to include the isospin dependence of the in-medium nucleon-nucleon cross-section by Li *et al.* [7], and is in good agreement with the present study. The shaded region in the figure corresponds to that obtained by constraining the binding energy, neutron skin thickness and isospin analogue state in finite nuclei using the mass formula of Danielewicz [8]. The yellow solid curve corresponds to the parameterization adopted by Heiselberg *et al.* [9] in their studies on neutron stars. By fitting earlier predictions of the variational calculations by Akmal *et al.* [10], where the many-body and special relativistic corrections are progressively incorporated, Heiselberg and Hjorth-Jensen obtained a value of C^{o}_{sym} = 32 MeV and γ = 0.6, similar to those obtained from the present measurements. A similar result is also obtained from the relativistic Dirac-Brueckner calculation, with C^{o}_{sym} = 32.9 MeV and γ = 0.59 [11]. The Dirac-Brueckner is an "ab-initio" calculation based on nucleon-nucleon interaction with Bonn A type potential instead of the AV18 potential used in the variational calculation of Ref. [10]. The density dependence of the symmetry energy has also been studied in the framework of an expanding emitting source (EES) model by Tsang *et al.* [12], where a power law dependence of the form $C_{sym}(\rho) = 23.4(\rho/\rho_o)^{\gamma}$, with $\gamma = 0.6$, was obtained. This dependence (shown by the pink dashed curve) is significantly softer than other dependences shown in the figure. The pink square point in the figure correspond to the value of symmetry energy obtained by fitting the experimental differential cross-section data in a charge exchange reaction using the isospin dependent CDM3Y6 interaction of the optical potential by Khoa *et al.* [13].

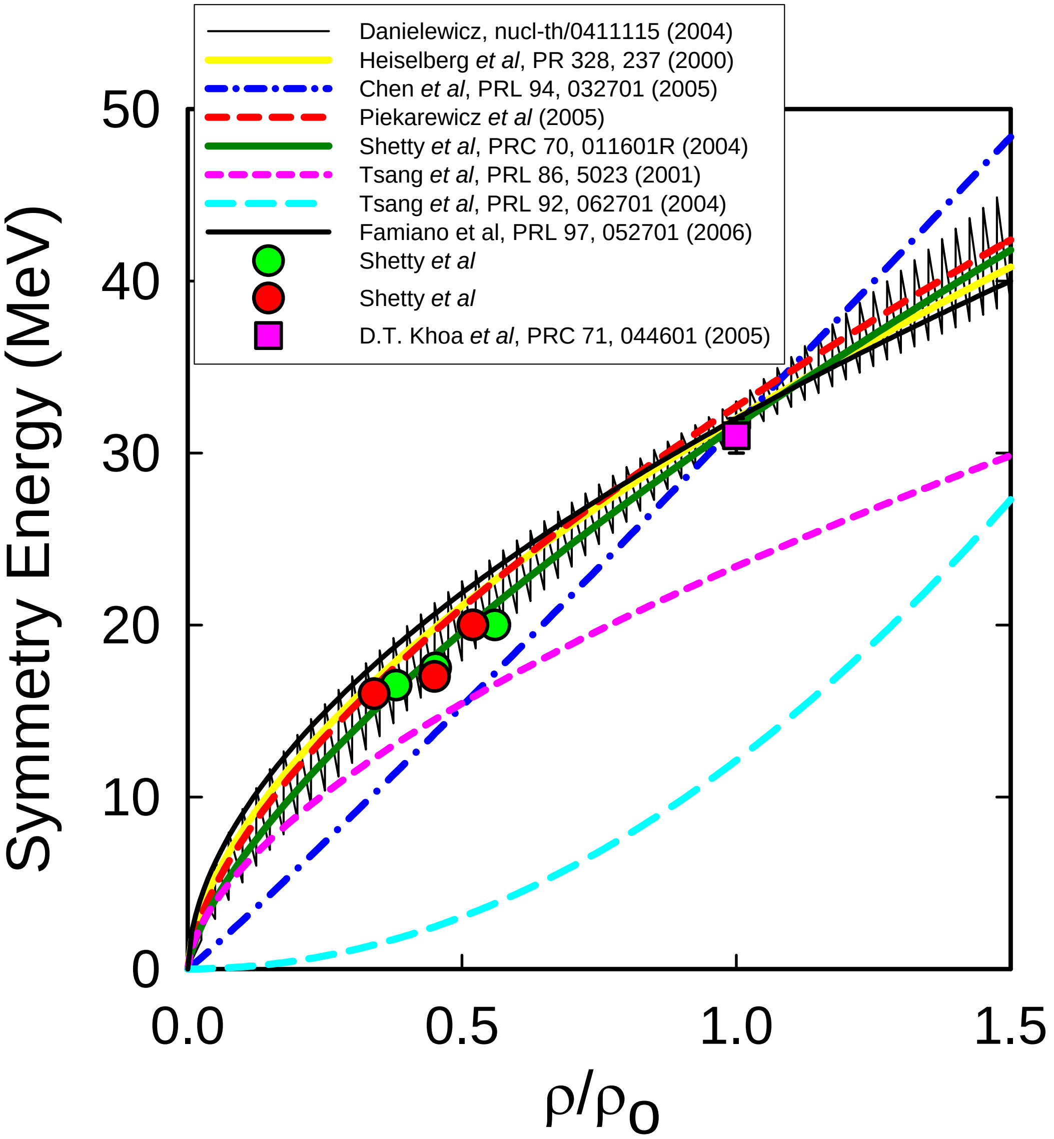

Figure 1. Comparison between the results on the density dependence of the symmetry energy obtained from various different studies. The different curves and symbols are described in the

An alternate observable, the double neutron/proton ratio of nucleons taken from two reaction systems using four isotopes of the same element, has recently been proposed as a probe to study the density dependence of the symmetry energy [14]. This observable is expected to be more robust than the isoscaling observable. It was shown recently [14] that the experimentally determined double-ratio for the ^{124}Sn + ^{124}Sn reaction to that for the ^{112}Sn + ^{112}Sn reaction, results in a dependence with $\gamma = 0.5$ (shown by black solid curve), when compared to the predictions of the IBUU transport model calculations. This observation is in close agreement with other studies discussed above. However, this dependence has been obtained by using the momentum independent calculation of Ref. [15]. A more recent calculation [16] using a BUU transport model that includes momentum dependent interaction shows significantly lower values for the double neutron/proton ratio of free nucleons compared to the one reported by Famiano *et al.*

The parameterized form of the density dependence of the symmetry energy obtained from all the above studies are as shown in Table I. The close agreement between various independent studies therefore suggests that a constraint on the density dependence of the symmetry energy, given as $C_{sym}(\rho) = C^{o}_{sym}(\rho/\rho_o)^{\gamma}$, where $C^{o}_{sym} \sim 31$ - 33 MeV and $\gamma \sim 0.55$ - 0.69 can be obtained. This rules out extremely "stiff" and "soft" dependences predicted by theory.

Table I. Parameterized form of the density dependence of the symmetry energy obtained from various independent studies

Reference	Parameterization	Studies
Fuchs *et al.* [11]	$\mathbf{32.9(\rho/\rho_o)^{0.59}}$	**Relativistic Dirac-Bruckner cal.**
Heiselberg *et al.* [9]	$\mathbf{32.0(\rho/\rho_o)^{0.60}}$	**Variational calculation**
Danielewicz *et al.* [8]	$\mathbf{31(33)(\rho/\rho_o)^{0.55(0.79)}}$	**BE, Skin,Isospin analogue states**
Tsang *et al.* [5]	$\mathbf{12.125(\rho/\rho_o)^{2}}$	**Isospin diffusion**
Chen *et al.* [6]	$\mathbf{31.6(\rho/\rho_o)^{1.05}}$	**Isospin diffusion**
Li *et al.* [7]	$\mathbf{31.6(\rho/\rho_o)^{0.69}}$	**Isospin diffusion**
Piekarewicz *et al.* [4]	$\mathbf{32.7(\rho/\rho_o)^{0.64}}$	**GMR, IVGDR**
Shetty *et al.* [1,2]	$\mathbf{31.6(\rho/\rho_o)^{0.69}}$	**Isotopic distribution**
Famiano *et al.* [14]	$\mathbf{32.0(\rho/\rho_o)^{0.55}}$	**Neutron-proton emission ratio**
Tsang *et al.* [12]	$\mathbf{23.4(\rho/\rho_o)^{0.6}}$	**Isotopic distribution**

We draw following conclusions from the above comparisons :

1) *Assuming a negligibly small sequential decay effect, the form of the density dependence of the symmetry energy obtained from the dynamical model analysis is in good agreement with the one obtained from the statistical model analysis.*

2) *The result of the statistical model analysis is in good agreement with other independent studies.*

3) *The isoscaling parameter probes the property of infinite nuclear matter* : The symmetry energy obtained from the dynamical model analysis corresponds to the volume part of the symmetry energy as in infinite nuclear matter, whereas, the symmetry energy obtained from the statistical model analysis corresponds to the fragments that are finite and have a surface contribution. The similarity between the two can probably be understood in terms of the weakening of the surface symmetry free energy when the

fragments are being formed. During the density fluctuation in uniform low density matter, the fragments are not completely isolated and continue to interact with each other, resulting in a decrease in the surface contribution. Using the constraint obtained for the volume part of the symmetry energy from the present study, and following the expression for the symmetry energy of finite nuclei, the general expression for the density dependence of the symmetry energy can be given as,

$$S_A(\rho) = \alpha(\rho/\rho_o)^{\gamma}/[1 + (\alpha(\rho/\rho_o)^{\gamma}/\beta A^{1/3})]$$

Where $\alpha = C^{o}_{sym} = 31 - 33$ MeV, $\gamma = 0.55 - 0.69$ and $\alpha/\beta = 2.6 - 3.0$. The quantities α and β are the volume and the surface symmetry energy at normal nuclear density. The above equation reduces to the volume symmetry energy for infinite nuclear matter in the limit of A tending to infinity, and to the symmetry energy of finite nuclei for $\rho = \rho_o$. The ratio α/β, is related to the neutron skin thickness, the measurement of which should provide a tighter constraint on the surface-volume correlation.

4) *The density dependence of the symmetry energy obtained using the statistical model approach is consistent with other experimentally determined observables* : In the past, attempts have been made to study the density dependence of the symmetry energy by looking at specific observables and comparing them with predictions of the dynamical models. Such an approach attempts to explain the observable of interest without trying to simultaneously explain other properties, such as, the temperature, density and excitation of the fragmenting system. The present statistical model approach simultaneously explains the isoscaling parameter, caloric curve, and the density as a function of excitation energy to arrive at the density dependence of the symmetry energy.

5) *Symmetry energy determined from the present study is lower than that of normal nuclei* : The present statistical model analysis yields volume contribution of the symmetry energy of the order of 18 – 20 MeV at half the normal nuclear density.

6) *The obtained constraint on the density dependence of the symmetry energy has important implications for astrophysical and nuclear physics studies* :

   a) *Neutron skin thickness* : It has been shown that an empirical fit to a large number of mean field calculations yield neutron skin thickness for ^{208}Pb nucleus, $R_n - R_p \sim (0.22\,\gamma + 0.06)$ fm, where γ is the exponent that determines the stiffness of the density dependence of the symmetry energy. From the above comparison, one obtains a neutron skin thickness of 0.18 – 0.21 fm.

   b) *Neutron star mass and radius :* The constraint also predicts a limiting neutron star mass of M_{max} = 1.72 solar mass and a radius, R = 11 – 13 km for the "canonical" neutron star. Recent observations of pulsar-white dwarf binaries at the Arecibo observatory suggest a pulsar mass for PSRJ0751+1807 of M = 2.1(+0.4, –0.5) solar mass at a 95% confidence level.

   c) *Neutron star cooling* : The constraint obtained predicts a direct URCA cooling for neutron stars above 1.4 times the solar mass. In such a case the enhanced cooling of an M = 1.4 solar mass neutron star may provide strong evidence in favor of exotic matter in the core of a neutron star.

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