Self-consistent Calculations for Isoscalar Giant Resonances

B. K. Agrawal and S. Shlomo

Studies of isoscalar giant monopole and dipole resonances are of particular interest since the centroid of their strength function is directly related to the nuclear matter incompressibility coefficient K. Currently, Hartree-Fock (HF) based random phase approximation (RPA) calculation for isoscalar giant monopole resonance (ISGMR) reproduce the experimental data for effective interaction associated with incompressibility $\mathrm{K} = 210 \pm 20$ MeV. The centroid energies (E_1) of isoscalar giant dipole resonances (ISGDR) provides an independent source of information for the incompressibility coefficient K. We would like to remark here that, although not always stated in the literature, actual implementation of HF-RPA are not fully self-consistent. One usually makes one or more of the following approximations:

1. Neglecting the two-body Coulomb and spin-orbit term in particle-hole interaction V_{ph}
2. Approximation momentum dependent parts in V_{ph}
3. Limiting the $p-h$ space in a discretized calculation by a cut-off energy $E_{ph}^{\max}$
4. Introducing a smearing parameter (i.e., a Lorenzian width $\Gamma/2$)
5. Numerical accuracy

Each of these approximations induces spurious state mixing (SSM), and may significantly affect the ISGDR centroid energy. In this contribution we shall present fully self-consistent HF-RPA results for the ISGDR strength function and the transition density for the spurious state in case of ^{80}Zr. We also investigate the effects of some of the approximations as listed above.

To perform a fully self-consistent calculation, one starts with adopting a viable form for an effective nucleon-nucleon interaction, V_{12}, and carries out HF calculation for the ground state of the nucleus. Then, RPA calculation is done with V_{ph} derived from the interaction V_{12}. The RPA Green's function G can be expressed in terms of free Particle-hole Green's function G_0 in the matrix form as

$$G = G_0(1 + V_{ph}G_0)^{-1}. \quad (1)$$

For a scattering operator $F = \sum_{i=1}^{A} f(r_i)$, the strength function and the transition density are given by

$$S(E) = \frac{1}{\pi}\mathrm{Im}[Tr(fGf)], \quad (2)$$

$$\rho_t(r,E) = \frac{1}{\sqrt{S(E)}}\int f(r')[\frac{1}{\pi}\mathrm{Im}\,G(r',r)dr']. \quad (3)$$

To calculate $S(E)$ and $\rho_t(r,E)$ in case of ISGMR we use

$$f(r) = r^2 Y_{00}(\Omega)$$

and that for ISGDR we use

$$f(r) = r^3 Y_{1m}(\Omega).$$

To project out the spurious state mixing in ISGDR we use the operator

$$f_\eta(r) = (r^3 - \eta r)Y_{1m}(\Omega)$$

with

$$\eta = \frac{< f_3 \rho_{ss} >}{< f_1 \rho_{ss} >}.$$

where ρ_{ss} is the spurious state transition density.

In what follows, we present our results obtained with

$$V_{12} = \delta(r_1 - r_2)[t_0 + \frac{1}{6} t_3 \rho^\alpha (\frac{r_1 + r_2}{2})] \quad (4)$$

where $\alpha = 1$, t_0, and t_3 are taken to be -1100 MeVfm^3, and 16000 MeV fm$^{3(\alpha+1)}$, respectively. The single-particle continuum states are treated exactly. Thus, no approximations, as listed above, are made, and the calculation is fully self-consistent.

We obtain numerical solutions for HF equation over a grid (δr) of 0.04 fm in the coordinate space. We use the same grid size for the evaluation of the Green's function G (Eq. 1) and dimension, N_{RPA}, of the RPA matrix. $[1 + V_{ph} G_0]$ is taken to be 300.

We perform a fully self-consistent (smearing parameter $\Gamma = 0$) calculation for ISGDR strength function. We find that the strength function for the F_3 is exactly identical to that for the operator F_η. In Fig. 1 we display our results for S(E) in case of ^{80}Zr for $\Gamma = 0.5$ MeV. We see that with the strength function for F_3 and F_η is practically the same. For $\Gamma = 2.0$ MeV (not shown here), we find that, due to SSM, the F_3 and F_η response are noticeably different for $E \leq 30$ MeV.

From the response for F_1 we find the position of spurious peak (E_{ss}) is 0.09 MeV. It is worth mentioning that these results do not change at all even if the calculations are repeated with quadrupole precision instead of double precision as used above.

In Fig. 2 we compare collective transition density $\propto \delta\rho / \delta r$ with the corresponding one obtained microscopically using RPA. As can be seen from the figure, it is hard to distinguish between the microscopic and collective transition densities.

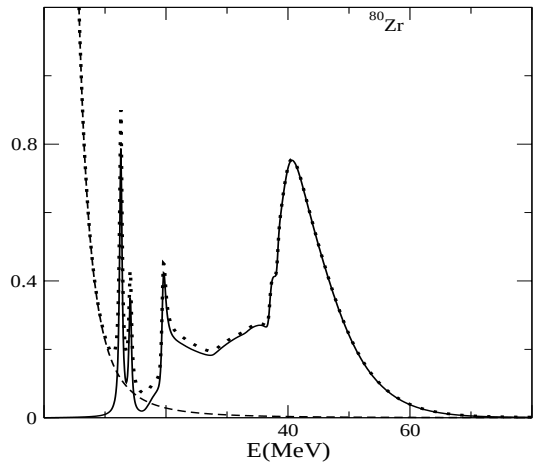

Figure 1: Self-consistent HF-RPA result of ISGDR strength function for F_3 (dotted), ηF_1 (long dashed) and F_η (solid line) in case ^{80}Zr nucleus.

In Table I we list the values of E_{ss}, E_0, and E_1 obtained for various combinations of the δr and N_{RPA}. We see from the table that the spurious state gets pushed away from zero

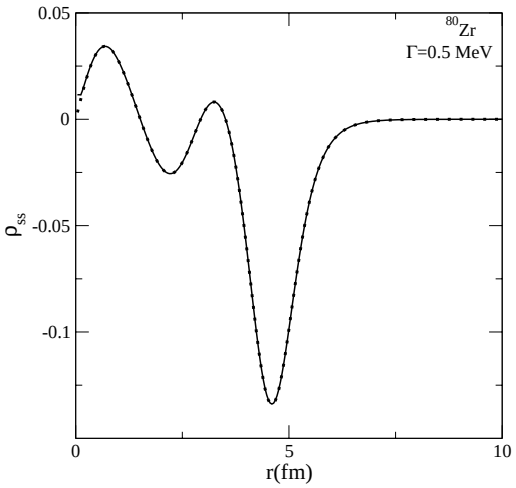

Figure 2: Comparison of the microscopic (dashed line) and collective (solid line) transition densities for the spurious state.

when various numerical approximations are made, leading to SSM. We infer from the table that the ISGMR and ISGDR do not change significantly for $E_{ss} \leq 1$ MeV.

Table I:

Γ	$(\delta r)_{HF}$	$(\delta r)_{RPA}$	N_{RPA}	E_{ss}	E_0	E_1
0	0.04	0.04	300	0.09	29.70	42.25
0	0.04	0.24	50	<0	29.57	42.07
0	0.08	0.08	150	0.19	29.62	42.21
0	0.08	0.24	50	<0	29.52	42.03
3	0.08	0.08	150	0.88	29.74	42.75
3	0.08	0.24	50	0.03	29.64	42.04

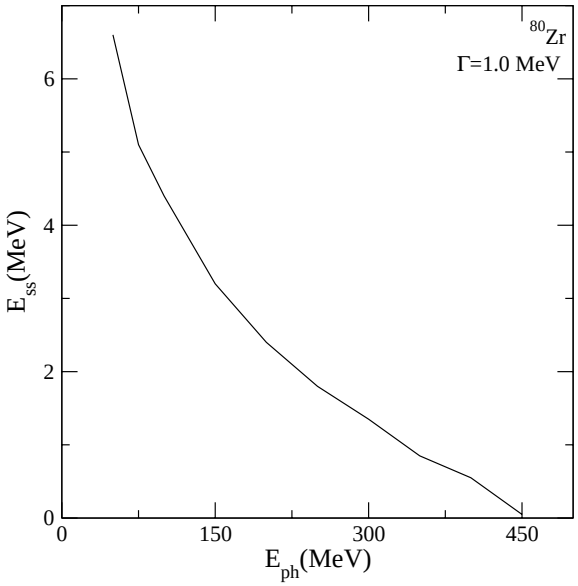

Figure 3: Variation of E_{ss} as a function of $E_{ph}^{\max}$.

References

[1] B. K. Agrawal, S. Shlomo, and A. I. Sanzhur, to be published.