Contribution of Vortex Motion to the Isoscalar Compression Modes

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We study the structure of the isoscalar giant dipole resonance (ISGDR) and its low-lying satellite within the extended fluid dynamic approach. The giant multipole resonances, which take up a main part of the energy-weighted sum rule (EWSR), can be successfully described using the irrotational fluid dynamics [1]. However, microscopic RPA analysis of flow patterns of the low-lying collective states shows strong rotational components. We show that inclusion of rotational (vortex) flow in the fluid dynamic approach generates the low-lying satellite of the ISGDR and leads to a decrease of energy of the main resonance.

Starting from the collisionless kinetic theory, we reduce the kinetic equation for the Wigner distribution function to the equation of motion for the displacement field $\vec{\chi}(\mathrm{r},t)$, assuming the following general form of $\vec{\chi}(\mathrm{r},t)$,

$$\vec{\chi}(\mathrm{r},t) = \nabla\phi(\mathbf{r},t) + \kappa\nabla \times \mathbf{A}(\mathbf{r},t), \quad (1)$$

where $\phi(\mathbf{r},t)$ and $\mathbf{A}(\mathbf{r},t)$ are the unknown scalar and vector fields and the coefficient κ determines the relative contribution in the case of vortex motion. We show that in the case of eigenvibrations with $\vec{\chi}\ (\mathbf{r},t) \sim \exp(-i\omega t)$, the displacement field satisfies the following equation of motion

$$m\omega^2\chi_\alpha = \nabla_\beta \delta P_{\alpha\beta}. \quad (2)$$

Here, $\delta P_{\alpha\beta}$ is the pressure tensor

$$\delta P_{\alpha\beta} = -\frac{K}{9}\ \nabla\cdot\vec{\chi}\delta_{\alpha\beta},$$
$$-\mu\left(\nabla_\alpha\chi_\beta + \nabla_\beta\chi_\alpha - \frac{2}{3}\nabla\cdot\vec{\chi}\delta_{\alpha\beta}\right), \quad (3)$$

K is the incompressibility and μ is the coupling constant which depends on the Landau's parameters F_l in the interaction amplitude. In the case of isotropic interaction amplitude, one has

$$\mu = (3/2)s^2\ ,_F\ [1-(1+F_0)/3s^2] \quad (4)$$

where $,_F$ is the Fermi energy and the dimensionless zero-sound velocity s is derived by the following Landau's dispersion equation

$$-\frac{1}{F_0} = 1 + \frac{s}{2}\ln\frac{s-1}{s+1}\cdot \quad (5)$$

The term multiplied by μ in Eq. (3) is due to the dynamical distortion of the Fermi surface. We point out that, in contrast to the traditional fluid dynamic approaches, Eqs. (2) with $\delta P_{\alpha\beta}$ from Eq. (3) does not imply any restriction on the multipolarity of the Fermi surface distortion. However, the solution of Eq. (2) requires the solution of the Landau's dispersion equation (5) for the dimensionless sound velocity s.

In the case of sharp nuclear surface, the scalar and vector fields in Eq. (1) take the forms

$$\phi(\mathbf{r}) = \nabla j_L(q_\parallel r)Y_{LM}(\Sigma_\mathbf{r}),$$

$$\mathbf{A}(\mathbf{r}) = \nabla\times j_L(q_\zeta r)\mathbf{Y}_{LLM}(\Sigma_\mathbf{r}).$$

Both wave numbers $q_{\|}$ and $q_{\perp}$ are connected by following relation $\mu q_{\perp}^2 = (K/9 + 4\mu/3)\, q_{\|}^2$. The wave numbers $q_{\|}$ and $q_{\perp}$ and the vorticity parameter κ in Eq. (1) can be evaluated using the boundary condition. The boundary condition implies that the force $F_{\alpha} = (r_{\beta}/r)\delta P_{\alpha\beta}$ acting on the free surface of the nucleus must vanish. This leads to the conditions

$$\mathbf{n} \theta \mathbf{F} *_{r = R_0} = 0,$$

$$\mathbf{n} \times \mathbf{F} *_{r = R_0} = 0, \qquad (6)$$

where R_0 is the radius of the nucleus and $\boldsymbol{n}$ is the unit vector normal to the nuclear surface.

We solved Eqs. (2) and (6) for the dipole mode L^{π} = 1^- and evaluated the eigenenergy $\hbar\omega$ and the vorticity parameter κ for a few lower excited states, including the ISGDR. The coupling constant μ was derived from Eqs. (4) and (5) using F_0=0.2. The lowest 1^- state occurs at zero energy $\hbar\omega_{spur} = 0$ and corresponds to the uniform translation of the nucleus. The energy weighted sum rule is mainly exhausted by the next two states which are the low-lying satellite (1^- pygmy resonance) and the isoscalar giant dipole resonance. The energy of both states, $\hbar\omega_{sat}$ and $\hbar\omega_{ISGDR}$, decreases as $A^{-1/3}$ with particle number A. However, the splitting energy

$$\Delta E = \hbar\omega_{ISGDR} - \hbar\omega_{sat}$$

varies slightly with A. We show that the contribution of the vortex motion is significant for both the ISGDR and its low-lying satellite. We point out also that the vorticity of the displacement field leads to an increase of the collective mass parameter of the ISGDR lowering its eigenenergy $\hbar\omega_{ISGDR}$.

References

[1] V. M. Kolomietz and S. Shlomo, Phys. Rev. C **61**, 064302 (2000).