Transition Densities for Isoscalar Giant Multipole Resonance and The Overtone

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We study the transition density and the structure of the strength function in the region of the isoscalar giant multipole resonance (ISGMR) and its overtone. Both the quantum Hartree-Fock (HF) random-phase-approximation (RPA) and the semiclassical fluid dynamics approaches are used. Special attention is paid to the compression modes with multipolarities L = 0, 1 and 2. One of our goals is to study the influence of the Fermi-surface distortion (FSD) on the formation of the ISGMR and its overtones.

Assuming the transition operator in the form of $F({\mathbf{r}_i}) = \sum_{i=1}^{A} f(\mathbf{r}_i)$ with $f(\mathbf{r}) = r^L Y_{L0}(\Sigma)$ for $L \ge 2$, $f(\mathbf{r}) = r^2 Y_{00}(\Sigma)$ for L = 0 and $f_0(\mathbf{r}) = (r^3 - 0r)Y_{10}(\Sigma)$ for L = 1, we have carried out HF-RPA calculations using the RPA Green's function *G* method in the discretized continuum with a Lorentzian smearing. The energy smeared RPA strength function $\widetilde{S}(E)$ and the corresponding transition density are obtained from

$$\widetilde{S}(E) = \frac{1}{\pi} \operatorname{Im}[Tr(fGf)], \qquad (1)$$

$$\rho_t(\mathbf{r}, E) = \frac{\Delta E}{\sqrt{\widetilde{S}(E)\Delta E}}$$
$$\int f(\mathbf{r}') \left[\frac{1}{\pi} \operatorname{Im} G(\mathbf{r}', \mathbf{r}, E)\right] d\mathbf{r}'.$$
(2)

Note that (2) is associated with the strength in the region of $E \pm E/2$ and is consistent with

$$\widetilde{S}(E) = \left| \int \rho_t(\mathbf{r}, E) f(\mathbf{r}) d\mathbf{r} \right|^2 / \Delta E \qquad (3)$$

The quantum strength function $\tilde{S}(E)$ was compared with the analogous one $S^{FLA}(E)$ obtained within the semiclassical Fermi-liquid approach (FLA)

$$S^{\mathsf{FLA}}(E) = \sum_{n} \left| \int d\mathbf{r} \rho_{\mathrm{tr},n}^{\mathsf{FLA}}(\mathbf{r}) f(\mathbf{r}) \right|^2 g_{\gamma}(E, E_n).$$
(4)

Here, $\rho_{tr,n}^{FLA}(\mathbf{r})$ is the FLA transition density

$$\frac{1-a\delta_{L1}}{q}\delta(R_0-r)j'_L(qR_0)\bigg]\rho_0Y_{L0}(\hat{r}),\quad(5)$$

where Δ_0 is the bulk density, R_0 is the equilibrium nuclear radius and parameter *a* is determined by the translation invariance condition in the case of the dipole compression mode and is given by $a = j_1(x) / xj'_1(x)$, $x = qR_0$. The wave number *q* is derived from the boundary conditions of the Fermi liquid drop model. The weighted function $g_l(E, E_n)$ in Eq. (4) is given by

$$g_{\ell}(E,E_n) = \frac{1}{\pi} \frac{\gamma_n}{\left(E - E_n\right)^2 + \gamma_n^2},$$
$$\gamma_n = \frac{1}{2} \frac{c_{\zeta}}{\hbar c_o^2} E_n^2,$$

where $(_n$ is the damping parameter due to the nuclear viscosity, $c_0 = \sqrt{K'/9m}$ is the zero sound velocity, K' is the incompressibility coefficient in the nuclear Fermi liquid, $c_{\zeta} = 4\zeta/3m\rho_0$ and . is the viscosity coefficient. Our numerical calculations show a good



Figure 1: Ratio of the centroid energies *E1/E0* of the ISGDR and the ISMGR obtained in the traditional liquid drop model (LDM), semiclassical FLA (4) and HF-RPA (1), models.

agreement between $\widetilde{S}(E)$ and $S^{\text{FLA}}(E)$ strength functions.

In Fig. 1 we show the ratio E1/E0 between the centroid of the ISGDR and the ISGMR, as a function of the mass number, obtained from $\tilde{S}(E)$ and from $S^{FLA}(E)$. The prediction for the ratio E1/E0 obtained within the traditional liquid drop model (LDM) is shown in Fig. 1 by the dotted line. The Fermi surface distortion effects shift up the quantum RPA and semiclassical FLA ratio E1/E0 with respect to the LDM prediction.

Using the transition operator in the form (for L = 0)

$$F(\{\mathbf{r}_i\}) = \sum_{i=1}^{A} (r_i^4 - \eta r_i^2) Y_{00}(\Omega_i)$$
 (6)

and fitting *0* by the requirement that ISGMR transition strength at the resonance energy vanishes we derive the "quantum" collective transition density for the overtone of the ISGMR and obtained



Figure 2: The quantum (macroscopic) transition density $\Delta_{tr}(\mathbf{r}) = \rho_{tr}^{macl}(\mathbf{r})$ (solid line) and the HF-RPA result (2) (dashed line) obtained for the overtone *L*=0 in ²⁰⁸Pb, at 35.2 MeV, in arbitrary units.

$$\rho_{\rm tr}^{\rm macr}(\mathbf{r}) \sim 2r^2 \left(5 + r\frac{d}{dr}\right) \rho_{\rm eq}(\mathbf{r}) - \eta \left(3 + r\frac{d}{dr}\right) \rho_{\rm eq}(\mathbf{r})$$
(7)

where $\Delta_{eq}(\mathbf{r})$ is the equilibrium particle density. We also calculated the strength function

$$S(k) = \left| \int d\mathbf{r} \rho_{\rm tr}(\mathbf{r}) j_L(kr) Y_{L0}(\Omega) \right|^2.$$
(8)

for both cases of the macroscopic transition density $\Delta_{tr}(\mathbf{r}) = \rho_{tr}^{macr}(\mathbf{r})$ and the fluid dynamic transition density $\Delta_{tr}(\mathbf{r}) = \rho_{tr,n}^{FLA}(\mathbf{r})$, Eq. (5), [1]. In Fig. 2 we compare the quantum (macroscopic) transition density $\Delta_{tr}(\mathbf{r}) = \rho_{tr}^{macr}(\mathbf{r})$ with the HF-RPA result (2) obtained for the overtone L=0 in ²⁰⁸Pb.

References

[1] S. Shlomo, A. I. Sanzhur and V. M. Kolomietz, to be published.