Isoscalar Giant Dipole Resonance and Nuclear Matter Incompressibility Coefficient

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Studies of compression modes of nuclei are of particular interest since their strength distributions, $S(E)$, are sensitive to the value of the nuclear matter incompressibility coefficient, K [1]. At present, Hartree-Fock (HF) based random-phase-approximation(RPA) calculations for the isoscalar giant monopole resonance (ISGMR) reproduce the experimental data for effective interactions associated with incompressibility $K = 210 \pm 20$ MeV.

The study of the isoscalar giant dipole resonance (ISGDR) is very important since this compression mode provides an independent source of information on K. Early experimental attempts to identify the ISGDR in ^{208}Pb resulted with a value of $E_1 \sim 21$ MeV for the centroid energy. Very recent and more accurate data on the ISGDR obtained at our Cyclotron Institute for a wide range of nuclei seems to indicate that the experimental values for E_1 are smaller than the corresponding HF-RPA results by 3–5 MeV. This discrepancy between theory and experiment raises doubts concerning the unambiguous extraction of K from energies of compression modes.

In this work we address this discrepancy between theory and experiment by examining the relation between the strength function $S(E)$ and the excitation cross section $\sigma(E)$ of the ISGDR, obtained by ∀-scattering. We emphasize that it is quite common in theoretical work on giant resonance to calculate $S(E)$ for a certain scattering operator F whereas in the analysis of experimental data of $\sigma(E)$ one carries out distorted-wave-Born-approximation (DWBA) calculations with a certain transition potential. Here we present results of accurate microscopic calculations for $S(E)$ and for $\sigma(E)$ with the folding model (FM) DWBA with transition densities $\Delta_t(\mathbf{r})$ obtained from HF-RPA calculations and suggest a simple explanation for the discrepancy between theory and experiment concerning the ISGDR.

In self-consistent HF-RPA calculation one starts by adopting specific effective nucleon-nucleon interaction, V_{12}, carries out the HF calculation for the ground state of the nucleus and then solves the RPA equation using the particle-hole (p-h) interaction V_{ph} which corresponds to V_{12}. The RPA Green's function G is obtained from

$$G = G_0 (1 + V_{ph} G_0)^{-1}, \qquad (1)$$

where G_0 is the free p-h Green's function. For

$$F = \sum_{i=1}^{A} f(\boldsymbol{r}_i), \qquad (2)$$

the strength function and transition density are given by

$$S(E) = \sum_n \left|\langle 0|F|n\rangle\right|^2 \delta(E - E_n) = \frac{1}{\pi}\,\mathrm{Im}[\mathrm{Tr}(fGf)], \qquad (3)$$

$$\rho_t(\boldsymbol{r}, E) = \frac{\Delta E}{\sqrt{S(E)\Delta E}} \times \int f(\boldsymbol{r}') \left[\frac{1}{\pi}\,\mathrm{Im}\, G(\boldsymbol{r}', \boldsymbol{r}, E)\right] d\boldsymbol{r}' \qquad (4)$$

Note that (4) is consistent with the strength in the region $E \pm)E/2$ and is consistent with

$$S(E) = \left| \int \rho_t(\boldsymbol{r}, E) f(\boldsymbol{r}) d\boldsymbol{r} \right|^2 / \Delta E. \qquad (5)$$

In fully self-consistent HF-RPA calculations, the spurious state (associated with the center of mass motion) $T = 0$, $L = 1$ appears at $E = 0$ and no spurious state mixing (SSM) in the ISGDR occurs. However, although not always stated in the literature, actual implementations of HF-RPA (and relativistic RPA) are not fully self-consistent. One usually makes the following approximations: (i) neglecting the two-body Coulomb and spin-orbit interactions in V_{ph}, (ii) approximating momentum parts in V_{ph}, (iii) limiting the p-h space in a discretized calculation by a cut-off energy $E_{ph}^{\max}$, and (iv) introducing a smearing parameter (i.e., a Lorentzian with э/2). Although the effect of these approximations on the centroid energies of giant resonances is small (less than 1 MeV), the effect on the ISGDR is quite serious since each of these approximations introduces a SSM in the ISGDR.

Recently [1,2] we have shown that in order to correct for the effects of SSM on $S(E)$ and the transition density we use the projection operator

$$F_\eta = \sum_{i=1}^{A} f_\eta(\boldsymbol{r_i}) = F - \eta F_1, \qquad (6)$$

With $f_0 = f - 0f_1 = (r^3 - 0r)Y_{1M}(\Sigma)$. The value of 0 associated with the coherent state transition density

$$\rho_{ss} = \frac{\partial \rho_0}{\partial r} Y_{1M}(\Omega), \qquad (7)$$

where Δ_0 (is the ground state density of the nucleus, is given by

$$\eta = \langle f_3 \rho_{ss} \rangle / \langle f_1 \rho_{ss} \rangle = \frac{5}{3} \langle r^2 \rangle \qquad (8)$$

To determine the transition density Δ_t for the ISGDR we use (4) with F_0 and obtain Δ_0 then project out the spurious term by making use of (7)

$$\rho_t(r) = \rho_\eta(r) - \alpha \rho_{ss},$$

$$\alpha = \langle f_1 \rho_n \rangle / \langle f_1 \rho_{ss} \rangle \qquad (9)$$

We have carried out [1] numerical calculations for the $S(E)$, $\Delta_t(\boldsymbol{r})$ and $\Phi(E)$ within the FM-DWBA-HF-RPA theory. We used the SLI Skyrme interaction, which is associated with $K = 230$ MeV, and carried out HF calculations using a spherical box of $R \geq 25$ fm. For the RPA calculations we used the Green's function approach with mesh size $)r = 0.3$ fm and p-h maximum energy of $E_{ph}^{\max} = 150$ MeV (we include particle states with principle quantum number up to 12), since it is well-known that in order to extract accurate $\Delta_t(\boldsymbol{r})$, $E_{ph}^{\max}$ should be much larger than the value required ($E_{ph}^{\max} \sim 50$ MeV) to recover EWSR. Since in our calculation we also neglected the two-body coulomb and spin-orbit interactions, the spurious state energies differ from 0 by a few MeV. We therefore renormalized the strength of the V_{ph} by a factor (0.99 and 0.974 for ^{116}Sn and ^{208}Pb, respectively), to place the spurious state at E=0.2 MeV. We have included a Lorentzian smearing (э/2 = 1 MeV) and corrected for the SSM as described above. We carried out the

FM-DWBA calculation for $\Phi(E)$ using a density dependent Gaussian nucleon-∀ interaction with parameters adjusted to reproduce the elastic cross section, with Δ_0 and Δ_t from HF-RPA.

Using the operator $f = r^2$ for the IS-GMR we calculated the corresponding $S(E)$, for E up to 60 MeV. We recover 100% of the corresponding EWSR and obtained the values of 17.09 and 14.48 MeV for the centroid energy of the ISGMR in ^{116}Sn and ^{208}Pb, respectively. The corresponding recent experimental values obtained at our Institute are 16.07 ± 0.12 and 14.17 ± 0.28 MeV, respectively.

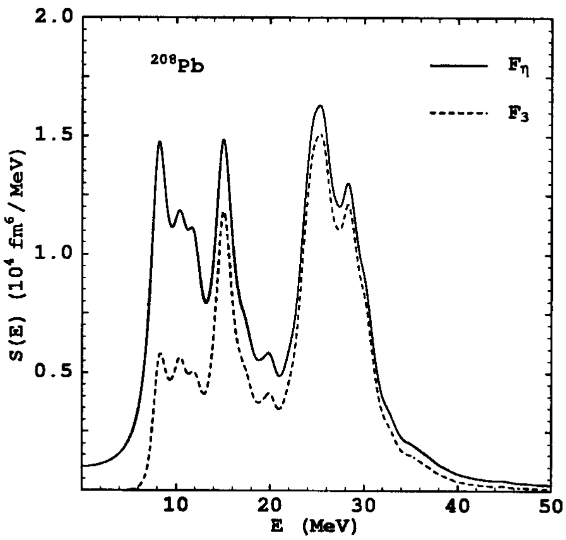

Figure 1: Strength functions for the ISGDR in ^{208}Pb obtained from Eqs. (4), (9) and (5), using f_3 (dashed line) and $f_\eta = f_3 - \eta f_1$ (solid line), with *0* = 52.1 fm^2.

Figure 1 exhibits the strength functions for the ISGDR in ^{208}Pb obtained from Eqs. (5), (4) and (9). The solid line describes the result obtained using f_0. Note that this result coincides with $S_0(E)$, which is free of SSM contribution. Similarly, the dashed line describes the erroneous result obtained using f_3 (it is also different from $S_3(E)$). We find that when using f_3, the excitation strengths obtained for certain states are sensitive to the value of э. The result obtained with f_3 coincides with that obtained with f_0 for э → 0, as expected. Thus, in configuration RPA calculation of Δ_t, one may use f_3 and correct for the SSM contribution before the smearing process.

Our results for the ISGDR, $S_0(E)$, indicate two main components with the low energy component containing close to 30% of the EWSR (for E up to 23 and 19 MeV for ^{116}Sn and ^{208}Pb, respectively), in agreement with the experimental observation [3].

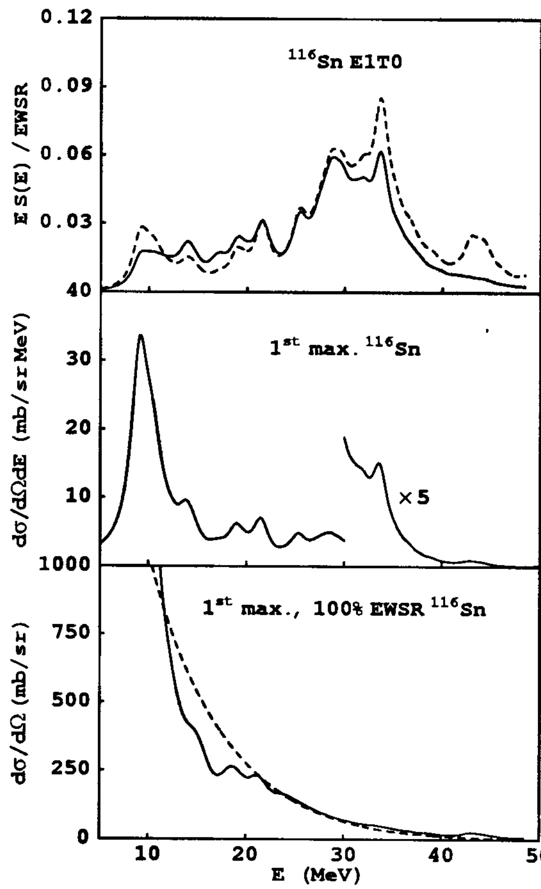

Figure 2: The ISDGR in ^{116}Sn. The middle panel: maximum double differential cross section obtained from Δ_t (RPA). The lower panel: maximum cross section obtained with Δ_{coll} (dashed line) and Δ_t (solid line) normalized to 100% of the EWSR. Upper panel: The solid and dashed lines are the ratios of the middle panel curve with the solid and dashed lines of the lower panel, respectively.

In Figure 2 we present results of microscopic calculations of the excitation cross section of the ISGDR in ^{116}Sn by 240 MeV ∀-particle, carried out within the FM-DWBA. The dashed lines are obtained using $\Delta_{coll}(r)$ of the ISGDR. It is seen from the upper panel that the use of Δ_{coll} increases the EWSR by at least 10% and may shift the centroid energy by a few percent. An important result of our calculation is that the maximum cross section for the ISGDR drops below the current experimental sensitivity of 2 mb/sr/MeV for excitation energy above 35 MeV (30 MeV for ^{208}Pb), which contains about 20% of the EWSR. This missing strength leads to a reduction of more than 2.5 MeV in the ISGDR energy and thus explains the discrepancy between theory and experiment. More sensitive experiments and/or with higher ∀-particle energy are thus needed.

In summary, we developed and applied an accurate and general method to eliminate the SSM contributions from $S(E)$ and Δ_t. Our results indicate: (i) Existence of non-negligible ISGDR strength at low energy and (ii) Accurate determination of the relation between $S(E)$ and $\Phi(E)$ resolves the long standing problem of the conflicting results obtained for K, deduced from experimental data $\Phi(E)$ for the ISGDR and data for the ISGMR.

References

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